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Cite as: AIP Advances **11**, 015208 (2021); https://doi.org/10.1063/9.0000045
Submitted: 14 October 2020 . Accepted: 25 November 2020 . Published Online: 05 January 2021

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Presented: 4 November 2020 • Submitted: 14 October 2020 •
Accepted: 25 November 2020 • Published Online: 5 January 2021

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Note: This paper was presented at the 65th Annual Conference on Magnetism and Magnetic Materials.
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ABSTRACT

We analyze spatially localized breather excitations for the model of a discrete Heisenberg spin chain, which includes the antisymmetric exchange interaction and single-ion anisotropy of the easy-plane type. In a finite size chain the breather modes may be indexed by number of embedded kink-antikink pairs forming the regular breather lattice. The influence of the antisymmetric exchange on properties of the discrete breather modes is examined.

I. INTRODUCTION

According to generally accepted definition, a breather is a localized in space and periodic in time solution of either continuous media equations or discrete lattice equations. The breather solutions were found, for example, for the exactly solvable sine-Gordon equation.[1–3] The discrete nonlinear lattices also reveal spatially localized oscillating modes. It was found out that nonlinearity and discreteness are two pivotal ingredients supporting these excitations, which were named as discrete breathers (DB) or intrinsic localized modes.[4,5] Until recently, the main attention in the study of the DBs was focused on crystal lattices,[6,7] although there is experimental evidence of their appearance in other physical systems: optical waveguides,[8] Bose–Einstein condensates,[9] granular crystals,[10] Josephson junctions.[11] However, there remains a need to find these excitations in magnetic media. A theoretical search for discrete breathers in spin systems was undertaken in Refs. 12–14, where possibility of their appearance has been demonstrated on the example of Heisenberg ferromagnetic and antiferromagnet spin chains with single-ion magnetic anisotropy. It has been confirmed that these intrinsic localized modes have high frequencies, above the maximum frequency of the spin-wave spectrum. Recently, the localized breather modes have been studied in a weak ferromagnetic spin lattice with the Dzyaloshinskii–Moryia (DM) interaction.[15]

In this paper we investigate localized breather modes in the model of monoaxial chiral helimagnet. Following the idea suggested in Ref. 12, we address the so-called phase of forced ferromagnetism, where in the ground state all spins align with an external magnetic field applied along the chiral axis.[16] Our analysis shows that the intrinsic localized breather modes do exist and we demonstrate how the DM interaction affects their properties.

This paper is organized as follows. In Sec. II, we describe the model and basic equations to search for discrete breathers. Their explicit form is given in Sec. III. The energies of these localized modes are derived in Sec. IV. The conclusions are set out in Sec. V.

II. THE MODEL

The model spin Hamiltonian of the chiral monoaxial helimagnet has the form[16]

$$\mathscr{H} = -2J\sum_n \mathbf{S}_n \cdot \mathbf{S}_{n+1} + A\sum_n \left(S_n^z\right)^2 - H_0\sum_n S_n^z + D\sum_n [\mathbf{S}_n \times \mathbf{S}_{n+1}]_z,$$

where $\mathbf{S}_n$ is the n-th site spin vector. The first term corresponds to exchange coupling along the z axis with $J > 0$. The second describes the single-ion anisotropy with $A > 0$, the third is the Zeeman coupling with an external magnetic field H_0 directed along the z axis. The last term corresponds to Dzyaloshinskii–Moryia interaction of the strength D. It is assumed that the magnetic field exceeds the threshold value $H_{cr} = 2S\left(\sqrt{4J^2 + D^2} - J + A\right)$ of the onset of the forced ferromagnetic state.

To find breather solutions we use the equation of motion for the raising spin operator S_n^+ ($S_n^\pm = S_n^x \pm iS_n^y$) that yields

$$\frac{i\hbar}{2JS}\frac{d}{dt}s_n^+ = \frac{H_0}{2JS}s_n^+ - 2Bs_n^+ s_n^z + s_n^+\left(s_{n+1}^z + s_{n-1}^z\right) - s_n^z\left(s_{n-1}^+ + s_{n+1}^+\right) + i\frac{D}{2J}s_n^z\left(s_{n-1}^+ - s_{n+1}^+\right), \quad (1)$$

where $B = A/2J$ and the normalized classical variables $s_n^\pm = S_n^\pm/S$, $s_n^z = \sqrt{1 - s_n^+ s_n^-}$ are introduced.

Using the substitutions $s_n^+(t) = s_n(t)\exp(-i\omega t + ikna)$, $s_n^z = \sqrt{1 - s_n^2}$ and separating real and imaginary parts, we obtain

$$\Omega s_n = -2Bs_n\sqrt{1 - s_n^2} + s_n\left(\sqrt{1 - s_{n+1}^2} + \sqrt{1 - s_{n-1}^2}\right) - \sqrt{1 - s_n^2}(s_{n-1} + s_{n+1})\sqrt{1 + \frac{D^2}{4J^2}}\cos(ka + \delta), \quad (2)$$

$$\frac{ds_n}{d\tau} = \sqrt{1 - s_n^2}(s_{n-1} - s_{n+1})\sqrt{1 + \frac{D^2}{4J^2}}\sin(ka + \delta), \quad (3)$$

where a is the lattice unit, k is the wave vector number, $\Omega = (\hbar\omega - H_0)/2JS$. The phase δ is conditioned by $\mathrm{tg}\,\delta = D/(2J)$, and the dimensionless time $\tau = t/t_0$ is introduced, where $t_0 = \hbar/(2JS)$.

III. DISCRETE BREATHERS

In the static case, when $ds_n/d\tau = 0$, Eq. (3) implies directly $ka = -\tan^{-1}(D/2J)$, meaning that the wave vector coincides with the counterpart of the simple or conical spiral, the ground states of the chiral helimagnet at $H < H_{cr}$.

The intrinsic localized modes have high frequencies, above the maximum frequency of the spin-wave spectrum.[12] Therefore, the discrete breathers are excited at the Brillouine zone boundary and characterized by a sign-changing ordering of s_n values. To develop analytical treatment, it is convenient to introduce the smooth envelope function $\psi(z) = (-1)^n s_n$, where $z = na$.

In the continuum limit, Eq. (2) is reduced to

$$\frac{d^2\psi}{d\tilde{z}^2} - \alpha\psi + \beta\psi^3 = 0, \quad (4)$$

if to neglect nonlinear terms including $d\psi/dz$ and $d^2\psi/dz^2$. Here, the coefficients are defined

$$\alpha = \frac{2B + \Omega - 2 - 2\sqrt{1 + \frac{D^2}{4J^2}}}{\sqrt{1 + \frac{D^2}{4J^2}}}, \quad \beta = \frac{B - 1 - \sqrt{1 + \frac{D^2}{4J^2}}}{\sqrt{1 + \frac{D^2}{4J^2}}} \quad (5)$$

and the dimensionless coordinate $\tilde{z} = z/a$ is introduced.

Solution of Eq. (4) may be presented in the form of the breather lattice

$$\psi(\tilde{z}) = \sqrt{\frac{\alpha + \sqrt{\alpha^2 + 2\beta c}}{\beta}}\,\mathrm{cn}\left[\left(\alpha^2 + 2\beta c\right)^{1/4}(\tilde{z} - \tilde{z}_0), \kappa\right], \quad (6)$$

where $\mathrm{cn}(\cdots)$ is the elliptic Jacobi function with the modulus

$$\kappa^2 = \frac{\alpha + \sqrt{\alpha^2 + 2\beta c}}{2\sqrt{\alpha^2 + 2\beta c}}. \quad (7)$$

Here, $\tilde{z}_0$ corresponds to the center of the periodic solution, which may be located either on a lattice site (so-called Sievers–Takeno's mode[5]) or between the sites (Page's modes[17]). The constant c specifies both the amplitude and the period $L_{\mathrm{br}} = 4K\left(\alpha^2 + 2\beta c\right)^{-\frac{1}{4}}$ of the breather lattice, where K is the elliptic integral of the first kind. Not only magnon density, but the topological charge

$$\mathcal{Q} = \frac{\Delta\varphi}{2\pi} = \frac{kL_{\mathrm{br}}}{2\pi} = -\frac{2K}{\pi(\alpha^2 + 2\beta c)^{1/4}}\tan^{-1}\left(\frac{D}{2J}\right), \quad (8)$$

is accumulated in the breather lattice due to Dzyaloshinskii–Moryia interaction. Here, $\Delta\varphi$ is the total angle of spin moment rotation around the z axis per period of the breather lattice.

The parameter c, which fixes the elliptic modulus for given α and β, determines a density of breathers per unit length for a system of infinite linear size. In particular, the breather lattice transforms into the single breather solution at $c = 0$, or equivalently $\kappa = 1$,

$$\psi(\tilde{z}) = \sqrt{\frac{2\alpha}{\beta}}\frac{1}{\cosh\left[\sqrt{\alpha}(\tilde{z} - \tilde{z}_0)\right]}, \quad (9)$$

which satisfies the boundary conditions $\psi(\pm\infty) = 0$ and $(d\psi/dz)|_{z=\pm\infty} = 0$. Obviously, the inequalities $\alpha > 0$ and $\beta > 0$ are needed for the localized solution to exist that imposes limits on the frequency $\Omega > 2 - 2B + 2\sqrt{1 + \frac{D^2}{4J^2}}$ and the easy-plane anisotropy constant $B > 1 + \sqrt{1 + \frac{D^2}{4J^2}}$. Thereby, experimental observation of the discrete breathers in the static regime requires a strong single-ion anisotropy of the order, or bigger than the exchange coupling just as has been found in $CsFeCl_3$ or some quasi-1D metal-organic compounds.[18] In known monoaxial chiral helimagnets, f.e. $Cr_{0.33}NbS_2$, the inverse situation, $B < 1$, is realized,

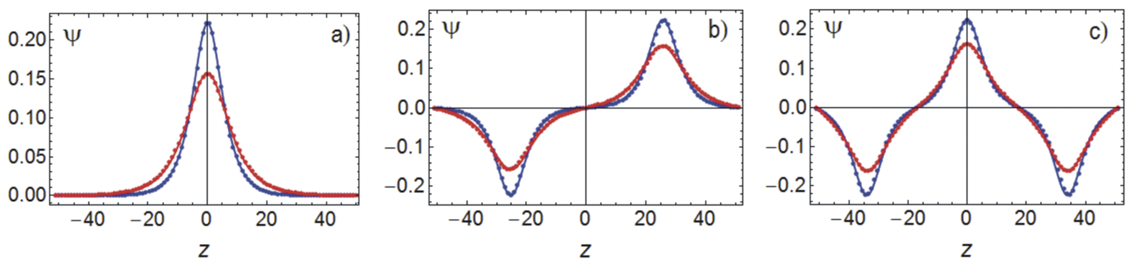

FIG. 1. The envelope function of breather excitations for the lattice of size L: (a) one kink-antikink pair (L = 100), (b) two kink-antikink pairs (L = 101), (c) three kink-antikink pairs (L = 100). The solid (dot) line corresponds to analytical (numerical) calculation for $D/J = 0$ (blue line) and for $D/J = 0.16$ (red line) with $B = 4.0$, $\Omega = -3.95$. The edge spin values are (a) $s_{-L/2} = 1.348 \cdot 10^{-6}$ (blue) and $s_{-L/2} = 1.967 \cdot 10^{-5}$ (red); (b) $s_{-L/2} = -3.270 \cdot 10^{-4}$ (blue) and $s_{-L/2} = -9.159 \cdot 10^{-4}$ (red); (c) $s_{-L/2} = 0.002177$ (blue) and $s_{-L/2} = 0.00048823$ (red).

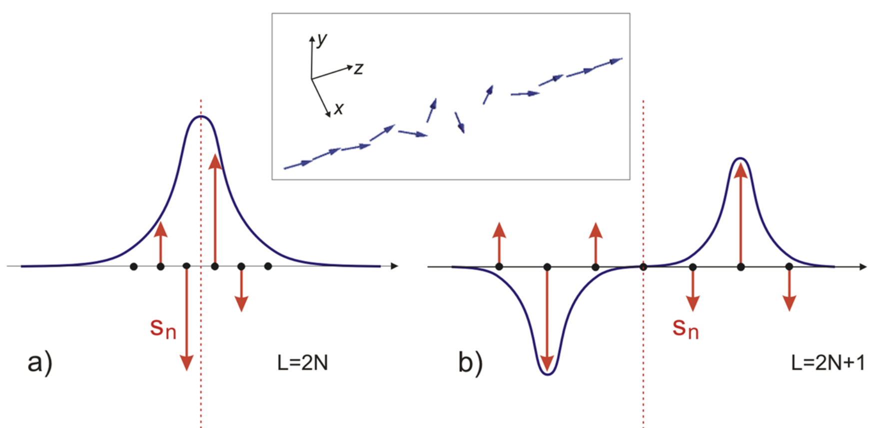

FIG. 2. Scheme of antisymmetric arrangement of spin variables s_n with respect to the chain center (red dashed line) in a presence of the DM interaction: (a) even and (b) odd number of lattice sites. The solid blue line shows a profile of the envelope function $\psi(z)$. Inset: spatial distribution of spins (blue arrows) inside the chain for the discrete breather with one kink-antikink pair.

which allows for travelling discrete breathers. A relevant analysis of this case will be given elsewhere.

For a finite size system the constant c may be derived from the condition that dynamics of the edge and interior spins must be consistent. To illustrate finding of c and κ, take the example of the finite size chain $(-L/2, L/2)$ centered at $z_0 = 0$ with the open boundary conditions. By definition, the constant c arises as the first integral of Eq. (4)

$$c = \left(\frac{d\psi}{d\tilde{z}}\right)^2 - \alpha\psi^2 + \frac{\beta}{2}\psi^4. \tag{10}$$

On the other hand, the derivative of the envelope function at the chain edge is given by

$$\left(\frac{d\psi}{d\tilde{z}}\right)\bigg|_{L/2} \approx \frac{1}{R}(1 + R - \Omega - 2B)\psi(L/2) + \frac{1}{2R}(2B - 1 - R)\psi^3(L/2), \tag{11}$$

which directly follows from Eq. (2), if to account the open boundary conditions and neglect nonlinear terms including derivatives of the function ψ. The notation $R = \sqrt{1 + \frac{D^2}{4J^2}}$ is introduced for brevity.

It can be envisaged that the envelope function is small at edges of the chain due to intrinsic localization of breathers. Then, Eqs. (11) and (10) result in the transcendental equation $c = \left[\alpha^2 + \alpha(R+2)/R + (R+1)^2/R^2\right]\psi(L/2)^2$. Together with Eq. (7), it allows to find c and κ depending on α and β. Figure 1 presents the profiles of the envelope function for the lowest breather excitations computed both analytically from Eq. (6) and numerically from Eqs. (2) and (3). It is clear that DM interaction suppresses the amplitude of the breather modes without preventing their occurrence. Numerical study also demonstrates that spatial distribution of the spin variables s_n is antisymmetric with respect to the chain center in the presence of the DM interaction (symmetrical solutions are also possible without the DM term). The center may be located either between the lattice sites (Page's mode) in the case of their even number, $L = 2N$, or directly on a site in the case of $L = 2N + 1$ (Takeno–Sievers mode). In the first case (see Fig. 2a), the breather mode contains odd number of kink-antikink pairs with a maximum of the envelope function at the center, while the breather modes include even number of these pairs with a zero value of ψ at the center for the odd number of lattice sites (Fig. 2b).

IV. ENERGY OF THE DISCRETE BREATHERS

Using the envelope function, the energy of the breather lattice takes the form

$$E - E_0 = \sqrt{4J^2 + D^2}S^2 \sum_n \psi(na)\psi(na + a) + \left(2JS^2 + \frac{1}{2}H_0 S - AS^2\right)\sum_n \psi^2(na), \tag{12}$$

where $E_0 = -2JS^2N - H_0SN + AS^2N$ is the energy of the ferromagnetic background. In the continuum approximation, the energy of the breather lattice per unit length may be calculated explicitly that leads to the result

$$\frac{E - E_0}{L_{\text{br}}} = \frac{\alpha + \gamma}{\beta}\left(\frac{E}{K\kappa^2} - \frac{\kappa'^2}{\kappa^2}\right)\omega_0, \tag{13}$$

where $\gamma = \sqrt{\alpha^2 + 2\beta c}$, $\omega_0 = \left(\sqrt{4J^2 + D^2}S^2 + 2JS^2 + H_0S/2 - AS^2\right)$ and $\kappa'^2 = 1 - \kappa^2$ is the complementary elliptic modulus. This energy exhibits a linear dependence on number of bound kink-antikink pairs, n, that means that the n-th order breather solution presents a state with n unbound particles (bosons).[19] The energy plot (Fig. 3) shows the linear dependences both for $D/J = 0$ and $D/J = 0.16$ with a smaller slope in the last case due to smaller amplitude of breather solutions with nonzero DM interaction.

Besides the continuum part of the energy density, there is an additional pinning energy resulting from underlying discreteness of the system.[20] This is essential for an infinite size system with a single breather, when both the Takeno–Sievers and the Page's modes are possible simultaneously. We discuss the case in details.

It is convenient to indicate explicitly the dependence on z_0 in the solution (9) when calculating the energy (12)

$$\Delta E = E - E_0 = \int_{-\infty}^{+\infty} dz\left[\sum_n \delta(z - na)\right] \times \left[\sqrt{4J^2 + D^2}S^2\, \psi(z - z_0)\psi(z - z_0 + a) + \left(2JS^2 + \frac{1}{2}H_0S - AS^2\right)\psi^2(z - z_0)\right]. \tag{14}$$

The series $\sum_n \delta(z - na) = 1/a + (2/a)\sum_{n=1}^{\infty} \cos(2\pi nz/a)$ can be used to present the energy of a discrete breather as

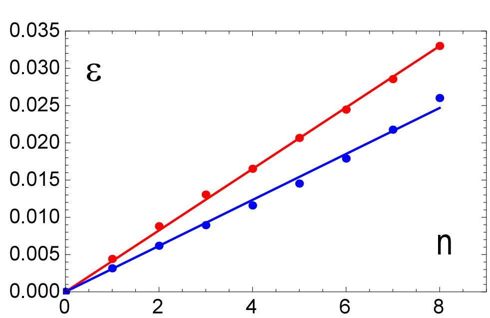

FIG. 3. The energy density of breather excitations for the chain of the length $L = 101$, $\varepsilon = (E - E_0)/\omega_0 L_{\text{br}}$, as a function of kink-antikink pairs number at $D/J = 0$ (red points) and $D/J = 0.16$ (blue points). The solid lines are linear interpolation of the numerical data.

$$\Delta E = \Delta E_0 + 2\sum_{n=1}^{\infty}\left[\sqrt{4J^2 + D^2}S^2 I_{1n} + \left(2JS^2 + \frac{1}{2}H_0S - AS^2\right)I_{2n}\right],$$

where the part $\Delta E_0 = 4\sqrt{\alpha}\omega_0/\beta$ corresponds to continuum contribution into the breather energy, while the sum is nothing but the pinning energy, where

$$I_{1n} = \frac{4\pi^2 n}{\beta \sinh\left(\frac{\pi^2 n}{\sqrt{\alpha}}\right)}\left[\cos\left(\frac{2\pi n z_0}{a}\right) + n \sin\left(\frac{2\pi n z_0}{a}\right)\right],$$

and $I_{2n} = 4\pi^2 n \cos\left(\frac{2\pi n z_0}{a}\right)/\beta \sinh\left(\frac{\pi^2 n}{\sqrt{\alpha}}\right)$.

Choosing $z_0 = 0$ for the Sievers–Takeno mode (ST) or $z_0 = a/2$ for the Page's mode (P) we obtain the corresponding pinning energies $\Delta E_{\text{pin}}^{(\text{ST})} = 2\sum_{n=1}^{\infty}\omega_0 I_n$ and $\Delta E_{\text{pin}}^{(\text{P})} = 2\sum_{n=1}^{\infty}(-1)^n\omega_0 I_n$ with $I_n = 4\pi^2 n/\beta \sinh(\pi^2 n/\sqrt{\alpha})$. Obviously, the terms with large n contributes exponentially small additions to the pinning energy. Thus, the ST- and P-modes are separated by the energy barrier

$$\frac{\Delta E_{\text{pin}}^{(\text{ST})} - \Delta E_{\text{pin}}^{(\text{P})}}{\Delta E_0} \approx \frac{4\pi^2}{\sqrt{\alpha}\sinh\left(\frac{\pi^2}{\sqrt{\alpha}}\right)}.$$

V. CONCLUSIONS

We investigate discrete breather modes in the phase of forced ferromagnetism of the monoaxial chiral helimagnet within the model of a spin chain with the antisymmetric exchange interaction. The localized breather modes in a finite-size system may be categorized by number of embedded kink-antikink pairs forming the breather lattice. It is found that the DM interaction suppresses amplitude of these modes and lowers their symmetry by allowing only excitations with the odd envelope function with respect to the center of the chain. The energy of the discrete breather modes exhibits a linear dependence on number of bound kink-antikink pairs.

ACKNOWLEDGMENTS

The authors are grateful to S.V. Dmitriev for valuable discussions. This work was supported by the RFBR grant (project No. 20-02-00213), the Act of the Government of the Russian Federation (contracts No. 02.A03.21.0011 and No. 02.A03.21.0006), the Ministry of Science and Higher Education of the Russian Federation, project No. FEUZ-2020-0054.

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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